

Assessing children's strategy choices to make better decisions about remediation

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A considerable number of children rely on counting to solve single-digit addition problems when they are expected to use accurate retrieval-based strategies. There are different reasons why this may be so. Children may use inefficient counting strategies, produce errors when applying backup strategies or lack sufficient confidence to just state the answer. In this study, children in Years 2-6 ($n=94$) were assessed on how they solved single-digit problems. Data were analysed to identify five performance groups that represented different patterns of difficulty. The findings highlight how interventions need to be better tailored to suit individual learning needs and indicate how this may be achieved.

Developing skill with single-digit addition is a focus of instruction in first, second and third year. During this time, children generally progress from solving single-digit addition problems using a counting-all strategy to solving them using an efficient min-counting strategy, where the smaller addend is counted on from larger addend, and retrieval-based strategies (Hopkins & Lawson, 2002). Retrieval-based strategies encompass retrieval, where the answer is directly retrieved from memory and decomposition strategies, where an answer is derived using a known fact (e.g., $6+7=6+6+1$). By third year, curriculum documents suggest that children will predominantly use retrieval or retrieval-based strategies to solve single-digit addition problems (e.g., Australian Curriculum, Assessment and Reporting Authority, 2010).

Problem-solving practice, defined as repeated opportunities for solving problems using strategies of choice, plays a critical role in developing retrieval-based strategies for solving single-digit addition problems. Problem-solving practice with accurate backup strategies (any strategy other than retrieval) leads to (i) the increased use of more efficient backup strategies, as attentional resources are redeployed and more efficient backup strategies are discovered, (ii) the increased use of retrieval, as each time a problem is correctly solved using a backup strategy, an association between the problem and answer is strengthened in memory to increase the likelihood of retrieval, and (iii) the eventual dominance of retrieval, as practice leads to retrieval and practice with retrieval further increases the likelihood of retrieval (Siegler & Shrager, 1984; Shrager & Siegler, 1998). Many children discover efficient strategies for themselves and come to rely on retrieval-based strategies as a result of problem-solving practice (Canobi, 2009; Goldman, Pellegrino, & Mertz, 1988; Siegler & Jenkins, 1989). It should be noted that problem-solving practice is not the only approach used in classrooms for developing single-digit addition skill – it is usually complimented with instruction designed to foster an understanding of number and operations, along with exposure to more efficient backup strategies – but it is a prominent approach used in classrooms.

Not all children benefit from problem-solving practice to develop retrieval-based strategies for solving single-digit addition problems. For example, children with pronounced mathematics learning difficulties or mathematics learning disability (MLD) are often delayed in making the transition from using inefficient counting strategies to min-counting (Geary, Bow-Thomas, & Yao, 1992) and delayed in making the transition from

using min-counting to retrieval (Torbeys, Verschaffel & Gesquière, 2004). Others have argued that children with MLD have a retrieval deficit associated with problems with working memory (e.g., Geary, Hoard and Bailey, 2012). Based on these and similar findings, Fuchs, Powell, Seethaler, Fuchs and Hamlet (2010) recommended that interventions for children with MLD should encompass more direct teaching approaches like explicit instruction with min-counting as well as fact-retrieval practice (referred to as remedial drill and practice). Fact-retrieval involves children repeatedly seeing, hearing and/or saying problems with their corresponding correct answers and should not be confused with problem-solving practice. Both fact-retrieval practice and problem-solving practice can lead to an increased use of retrieval but fact-retrieval practice relies on memorising facts and problem-solving practice relies on experience solving problems to build problem-answer associations in memory.

Children are often selected to participate in intervention studies for improving single-digit addition skill on the basis that they exhibit MLD (i.e., score below some cut-off criterion on a standardised test of maths achievement). For example, children with MLD were selected for interventions incorporating fact-retrieval practice (e.g., Fuchs et al., 2006; Powell, Fuchs, Fuchs, Cirino, & Fletcher, 2009); Poncy, Skinner, & Jaspers, 2007) explicit instruction in min-counting (Fuchs, Powell, Seethaler, Cirino, et al., 2010), and more general numeracy interventions (e.g., Bryant et al., 2011).

Children with MLD comprise a heterogeneous group of learners (Bartelet, Dimona, Ansari, Vaesson, & Blomert, 2014) and yet existing intervention research assumes they are all the same and have the same learning needs. A more tailored approach to intervention research in the field is needed. Children selected for interventions relating to single-digit addition (for example) should first be assessed on how they solve single-digit addition problems - not just to establish that they are having difficulties with this particular skill but also because a well-designed assessment may reveal different reasons for why they are having difficulties.

The aim of this study was to assess the usefulness of a technique for identifying groups of children who display specific difficulties with simple addition. Consistent with the distributions-of-associations (DOA) model (Siegler & Shrager, 1984; Shrager & Siegler, 1998), children will be hindered in developing accurate retrieval-based strategies if they make frequent errors using backup strategies, or have not developed sufficient confidence to use retrieval due to setting a high (restrictive) confidence criteria for retrieval. This later difficulty is exemplified by a high reliance on efficient backup strategies and very few errors (Hopkins & Bayliss, 2014; Bailey, Littlefield, & Geary, 2012). The technique encompassed assessing children at one point in time and using a combination of cut-off scores and cluster analysis to delineate between five groups of children: (i) children who were proficient with single-digit addition, (ii) children who made some errors but were generally on track in terms of developing proficiency, (iii) children who displayed retrieval difficulties as a result of frequent min-counting errors, (iv) children who displayed retrieval difficulties as a result of a restrictive confidence criterion, and (v) children who frequently used inefficient counting strategies. The usefulness of the technique was assessed in terms of its potential to identify meaningful groups that might help advance intervention research in the field and, at the same time, highlight to teachers some of the performance characteristics they need to be aware of when deciding how to address individual learning needs.

Method

Setting

Participants attended a primary school located in Melbourne, with a school population of 427 children. The participating school has a lower than average value on the Index of Community Socio-Educational Advantage (ICSEA) compared to other schools in Australia. This index is based on indicators of children's family backgrounds, geographic location and the proportion of indigenous children in the school. At the time of the study, results from the National Assessment Program for Literacy and Numeracy (NAPLAN) indicated that Year 3 children in the school showed above average mathematics achievement compared to Year 3 children in like schools (i.e., schools with similar levels of educational advantage) and Year 5 children showed close to average achievement compared to Year 5 children in like schools.

Procedure

Initially all children in Year 2 (7-8 year olds) to Year 6 (11-12 year olds) were invited to take part in the study; written consent was obtained from 94 children and their parents (representing 33% acceptance rate). The sample included children in Year 2 (n=14), Year 3 (n=20), Year 4 (n=24), Year 5 (n= 15) and Year 6 (n=21). Each child was assessed on how they solved all 36 single-digit problems written in the form $x+y=$ (where $x \leq y$, and $1 < x, y \leq 9$). Performance was assessed on a problem-by-problem basis in terms of accuracy, response time (RT) and strategy use. To do this, each child was individually withdrawn from the classroom to a quiet room located nearby, for about 20 minutes. The child sat alongside the research assistant (RA), who was known to children in the school, and in front of a computer screen. When the child was ready, the RA pressed the space bar on the computer and a single-digit addition problem was randomly selected from the set and displayed. As soon as the child stated the answer out aloud, the RA pressed the space bar again and the child was prompted to type in his/her answer. The space bar on the computer activated a timer when the problem was presented and stopped the timer when the answer was called out. After giving the answer, the child was prompted to explain how they had calculated the answer and the strategy was recorded. The RA checked to see if the self-report was consistent with what she had observed. If the child could not remember the strategy they had used or the report conflicted with what had been observed, the strategy was recorded as being unclassified. The RA then tapped the space again and the next problem was displayed. This procedure was followed until the problem set had been completed. No feedback on accuracy was given. To begin with, the RA explained how the assessment worked and the child practiced the procedure with five problems (not included in the problem set). The procedure used in this study to identify strategy use on a trial-by-trial basis using a combination of self-report and observation, is commonly used in studies investigating single-digit addition skill (e.g., Canobi, 2009). Reaction times (RTs) were used in this study to investigate the validity of self-reports given (see results section).

Data from the initial assessment was used to group children according to how they solved single-digit addition problems. Children who never used a counting strategy were first identified and they formed the proficient group. Children who used a counting-all and/or a counting-from-right strategy on more than 33% of trials were identified next and they formed the inefficient counting group. K-means cluster analysis was used (Jain, 2010) to classify the remaining children into three groups, following the procedure

described by Hopkins and Bayliss (2014): children who used a mixture of min-counting and retrieval and were mostly accurate formed the almost-proficient group, children who were frequently inaccurate with min-counting formed the inaccurate min-counting group, and children who predominately used min-counting but made few mistakes formed the accurate min-counting group. Three measures were used in the cluster analysis: percentage use of retrieval, percent correct on retrieval trials and percent correct on min-counting trials.

Results

During the assessment with 94 children from Years 2-6, six types of strategies were reported: (i) counting-all, which involves one round of counting starting at one; (ii) counting-on-right, a counting-on strategy where the second addend is counted on the first addend; (iii) min-counting, another counting-on strategy where the smaller addend is counted on the larger addend, (iv) decomposition, where addends are partitioned to make use of a known fact (e.g., double facts or add-to-10 facts), (v) other, where other maths skills are applied to derive the answer (such as skip counting, compensation and multiplication), and (vi) retrieval, where the answer is directly retrieved from memory and is usually accompanied by an explanation similar to “I just knew it”. The strategies are summarized in Table 1, along with alternative labels sometimes used in the literature. Strategies were coded as being unclassified on 1.2% of trials.

Table 1
Strategies reported in the assessment

Label	Example	Alternative label
counting-all	$3+4=1, 2, 3, 4, 5, 6, \underline{7}$	sum-counting
counting-on-right	$3+6=3: 4, 5, 6, 7, 8, \underline{9}$	counting-on-from-first, max-counting
min-counting	$3+6=6: 7, 8, \underline{9}$	counting-on-from-larger
decomposition	$4+5=4+4$	thinking, derived-fact
other	$2+4=2+2+2$	skip counting strategy
	$3+3=2 \times 3$	multiplication strategy
	$3+5=4+4$	compensatory strategy
retrieval	7 (<i>just knew it</i>)	direct retrieval

Reaction times (RTs) were averaged for correct trials separated by strategy and the results were as follows: retrieval ($n=1735$, $M=2.3$, $SD=1.4$), decomposition ($n=106$, $M=5.4$, $SD=4.3$), other ($n=55$, $M=6.5$, $SD=5.8$), min-counting ($n=1074$, $M=6.4$, $SD=9.0$), counting-on-right ($n=90$, $M=10.8$, $SD=7.2$) and counting-all ($n=52$, $M=13.1$, $SD=7.2$). These findings support the validity of asking children to explain the strategy they had used immediately after solving the problem, as RTs were lowest on retrieval trials (around 2-3 seconds) and increased according to the efficiency of the strategy reported.

Data from the assessment was used to cluster children into five groups based on how frequently they used retrieval and inefficient counting strategies (i.e., counting-on-right and counting-all), and how accurate they were using retrieval and min-counting. Summaries of how children in these five groups solved single-digit addition problems are displayed in Table 2. Children in the proficient group solved the problem set predominately using retrieval and were mostly accurate. Children in the almost proficient

group used min-counting as frequently as those in the inaccurate min-counting group (around 27% of trials) but their performance was markedly different in terms of accuracy: 97% of min-counting trials were performed correctly by children in the almost proficient group compared to 59% by children in the inaccurate min-counting group). Children in the accurate min-counting group recorded the highest use of min-counting; these children predominantly used min-counting and were often accurate.

Table 2
Percentage for trials indicating strategy frequency and accuracy according to group

	Feature	Retrieval	Min-counting	Counting-on-right	Counting-all
Proficient (<i>n</i> =7)	Frequency	91.7	0	0	0
	Accuracy	98.8	-	-	-
Almost-proficient (<i>n</i> =35)	Frequency	67.7	26.1	1.1	0.1
	Accuracy	98.2	96.7	-	-
Inaccurate min-counting (<i>n</i> =8)	Frequency	58.3	27.4	3.5	2.4
	Accuracy	93.6	58.9	-	-
Accurate min-counting (<i>n</i> =39)	Frequency	35.5	54.8	1.9	0.4
	Accuracy	96.5	90.1	-	-
Inefficient counting (<i>n</i> =5)	Frequency	19.4	11.1	42.2	22.7
	Accuracy	92.2	77.4	79.7	93.8

Note. Accuracy is not shown if strategy use was less than 5%.

It should be noted that performance characterised by inaccurate min-counting and accurate min-counting appear to be distinct types of performance (rather than similar types of performance represented on a continuum). Children in the inaccurate group are less accurate with min-counting (59%) compared to the accurate min-accurate group (90%), but they use retrieval more often (58% compared to 36% respectively). These findings support the argument that children in the inaccurate min-counting group are hindered in their development of retrieval because they frequently make min-counting errors and children in the accurate min-counting group are hindered because of a high confidence threshold for retrieval.

It is also interesting to note that children in the inefficient counting group were mostly accurate using a counting-all strategy but were less accurate using a counting-on-right strategy (see Table 2). It might be expected that children would make more errors using a counting-all strategy, as they have more counts to make and so are more likely to lose track of the count, but in fact they made fewer errors. This finding could be explored further by looking at the error patterns for individual children in this group.

The numbers of children in each group, separated by year level, are displayed in Table 3. These figures indicate that children who frequently used inefficient strategies were mostly from Year 2, but children who frequently made min-counting errors or relied predominately on min-counting and made few errors, were found in the full range of year levels. The Australian Curriculum indicates that by the end of Year 3, children should be able to recall addition facts for single-digit numbers. These findings suggest that few children in this sample who were beyond Year 3 (less than 12%), were predominately using retrieval.

Table 3
Number of children in each group by year level

Group	Year 2	Year 3	Year 4	Year 5	Year 6
Proficient	0	0	4	2	1
Almost-proficient	1	7	8	7	12
Inaccurate min-counting	3	1	2	0	2
Accurate min-counting	6	12	9	6	6
Inefficient counters	4	0	1	0	0

Discussion

The aim of this study was to assess the usefulness of a technique for identifying groups of children who display specific difficulties with simple addition. To do this, 94 children from one school, in Years 2-6, were assessed on how they solved single-digit addition problems. Data were collected on a trial-by-trial basis and included self-reported strategy use, accuracy and RTs. Self-reports were validated using RT data.

The findings suggested that less than 12% of children in the school were using retrieval or retrieval-based strategies as often as would be expected, according to curriculum documents. This figure is consistent with other researchers (Cumming & Elkin, 1999; Geary, Hoard, Byrd-Craven & deSoto, 2004). This is a concern given the importance of learning retrieval-based strategies early in primary school. Geary (2011) found that the frequency of children's correct use of retrieval-based strategies to solve single-digit addition problems in first grade predicted growth in mathematics achievement through to fifth grade, even after controlling for domain-free cognitive abilities such as intelligence, working memory and processing speed. This is because retrieval-based strategies place less demand on working memory than counting strategies (Imbo & Vandierendonck, 2007), so children are able to focus on higher-order learning (Canobi, 2009; Carr & Alexeev, 2011).

The findings also revealed the potential usefulness of a prescribed technique for identifying groups of children who appeared to exhibit different patterns of difficulty. Some children were still using inefficient counting strategies, namely a counting-all strategy and a counting-on-right strategy. Interestingly, children who were counting-all were mostly accurate when using this strategy but children who had made the transition to using a counting-on-right strategy were often inaccurate. Some children in this study had made the transition to min-counting but were frequently inaccurate when using this strategy. It is important for teachers to observe and speak with children who frequently make errors to find out why they are making errors. It could be that children are losing track of the count or they might have developed a *procedural bug*. For example, Fuson (1985) noted that some children included the larger addend in the count as they were learning the min-counting strategy (e.g., $3+6=6, 7, 8$). Other children in this study had made the transition to min-counting and very accurate with this strategy, but had not made the transition to using retrieval. Shrager and Siegler (1988) explained that this pattern of performance was suggestive of a high confidence threshold for using retrieval. Siegler (1988) referred to it as a perfectionist pattern of performance. Bailey, Littlefield and Geary (2012) argued that more girls than boys exhibit a high confidence threshold for retrieval. The prevalence of this pattern of performance (referred to in this study as accurate min-counting) was high with around 40% of participants in this group. Further research is

needed to examine what factors influence confidence with retrieval and how confidence may be increased.

The importance of identifying groups of children with different performance profiles, which are linked to reasons that explain why they have difficulties with single-digit addition, will be greatly heightened if it could be shown that children with different profiles respond differently to the same intervention. This will be the crucial next step for investigating these groups. Based on the current approach in the literature, children are likely to receive instruction, like explicit instruction in min-counting or fact-retrieval practice, on the basis that they have a MLD. Findings from this study highlight the limitations of using such an approach. For example, it would be unwise to require children to memorise facts if they have constructed a procedural bug with counting. Likewise, some children who are not retrieving answers do not need to be taught min-counting but need to develop confidence with retrieval.

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